Mystery of the Infinite: Developing a Mathematically Based Summer Scholars Program

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Abstract

Since the summer of 2013, the Templeton Honors College, at Eastern University, has been conducting a Summer Scholars Program for high school students. In summer of 2014, two mathematicians were invited to design a mathematics based course for the Summer Scholars Program [5]. The resulting course included a rigorous study of discrete mathematics, as well as a variety of lectures and extracurricular activities integrating faith and philosophy with mathematics. This paper will give an overview of the program and the new course, highlighting successes as well as noting some of the challenges.

Introduction

Imagine teaching a college level Discrete Mathematics course in nine days. As if that were not challenging enough, imagine teaching the course to high school students, while also incorporating robust discussions of how faith and mathematics intersect. As crazy as it might sound, I helped design and implement a program doing just that in summer of 2014. In these pages, we will look at this program, beginning with an overview of the broader Summer Scholars Program at the Templeton Honors College. From there we will consider specifics of both the mathematics and the faith integration. I will share with you the outcomes, to include some of the student feedback. I will also highlight some of the challenges of designing and running such a program. Finally, we will look at how the program is continuing to develop. It is my hope that this material will be a useful resource to anyone interested in developing a similar program, and I invite you to share any ideas or wisdom that you might have.

Overview of the Program

Founded in 1999, the Templeton Honors College (THC) provides a home for undergraduate students at Eastern University who are interested in rigorous education that acknowledges the supremacy of Christ in all things. The college is named for Drs. John and Josephine Templeton, who have supported the college and its programs generously. The THC uses a great books curriculum, employs exemplary teachers who invite students into scholarship, and operates on a small cohort model.

In the summer of 2013, THC faculty and staff launched a new program, this time aimed at high school students. In its first summer, the Summer Scholars Program welcomed 28 high school students, from rising juniors to recent graduates, for a nine-day residential program hosted on Eastern's campus. The course was titled "The Examined Life: Knowledge, Wisdom, Virtue, Calling", and the program was designed to provide an authentic Templeton Honors College experience. Students read Plato's *Theaetetus*, Euripides *Bacchae* [4], and excerpts from Cicero's *Orator* [2], discussing the texts with THC faculty. Students were joined and assisted by current THC students, who served as TA's throughout the program. Successful students were able to earn three college credits, corresponding to a course offered at Eastern through the THC.

Throughout the course, students also had a chance to visit Independence Hall (Eastern is located in a Philadelphia suburb), to see a stage adaptation of C.S. Lewis' *The Great Divorce*, and to hear lectures from several other faculty speakers. Each day of the program began and ended with devotional time, and students were encouraged in everything they did to consider how the subjects they were studying related to their faith.

Students were enthusiastic about the program, with many students expressing interest in returning the following summer. As such, THC faculty and staff quickly began planning a new course that could be offered, setting on the theme "Citizenship: on Earth as it is in Heaven?" At the same time, they began to consider whether it would be possible to offer a second course, perhaps even a mathematics course.

Mystery of the Infinite - Specifics of the Course

As it turns out, one member of the THC faculty, Dr. Walter Huddell, is a mathematician. Dr. Huddell is also the chair of the Mathematics Department within Eastern's College of Arts and Sciences. When approached by THC leadership about developing a course for the Summer Scholars Program (SSP), Dr. Huddell was enthusiastic, and he soon invited me to join him in the work. We were given the task of adapting an existing mathematics course to the SSP format, and we settled on our Discrete Mathematics course.

At Eastern, Discrete Mathematics serves as a bridge course, giving students an introduction to proof writing and higher level mathematics. This choice, therefore, avoids topics that many students take in high school, while also avoiding Calculus as a prerequisite. We also recognized that the course offers a number of excellent opportunities for connecting the mathematics to philosophy and theology. As we explained some of the mathematics to the THC faculty and staff, they were particularly interested in the topic of cardinality, and we ultimately settled on "Mystery of the Infinite" as a title for the course (it was suggested that this would be better for advertising than "Discrete Mathematics").

Now, trying to fit a full semester of Discrete Mathematics into a 9 day program is not an easy task. What follows is a list of topics that we generally cover:

- Introductory Logic - Statements, Truth Tables, Logical Equivalences
- Techniques of Proof - Direct, Contradiction, Biconditional, Cases
- Set Theory - Basic Notation, Subsets, Set Operations, Proving Set Properties and Laws
- Relations - Definitions, Examples, Properties of Relations
- Functions - Definitions, Surjections, Injections, Bijections
- Cardinality - Countable and Uncountable Sets
- Equivalence Relations - Definitions, Quotient Sets, Modular Arithmetic
- Mathematical Induction

To even attempt to cover this amount of material, we recognized that the students would have to get started before they even arrived at Eastern. There was some precedent for this, as students in the 2013 SSP were expected to complete some reading and writing prior to the residential program. As such, we sent students a significant amount of material covering introductory logic and the basics of set theory. They then had about a month to read through the material and to complete a exercises based on the readings.

Once students arrived, we spent the first day of the program getting to know them and reviewing the pre-program material, answering any outstanding questions and trying to ensure that they had a good foundation for

rest of the course. From that point onward, we began to work through the rest of the material, generally giving three hour-long lectures each day. Each lecture was immediately followed by a problem session, recognizing that you can only truly learn mathematics by doing mathematics. Students also had evening problem sessions to continue processing the material they were learning.

During the course, students were expected to complete and submit homework on each topic that we covered, and we had TA's (again drawn from current Eastern students) grade the homework. We also gave two exams during the program, essentially corresponding to a midterm and a final. Finally, the students were able to visit the Museum of Mathematics in New York City (a welcome change of pace to the intensive lectures and problem sessions, and perhaps a bit more fitting for a mathematics course than a visit to Independence Hall).

Going Beyond the Mathematics

As with the other SSP courses, we ultimately wanted to help students deeply connect their faith to the subjects they were studying. Therefore, we built a number of other components into the program. In addition to some discussion of integration during lectures, each day began and ended with devotional time. Further, most evenings included a lecture or special activity that dealt explicitly with integrating faith into mathematics (or academics as a whole).

One of the highlights of the evening activities was a visit to Eastern's observatory and planetarium, where our resident cosmologist, Dr. David Bradstreet, discussed the integration of faith and science. On another night, students watched *The Proof* [3], which details Andrew Wiles journey in solving Fermat's Last Theorem; following the movie, students discussed themes with an Eastern alumnus. On other nights we invited theologians and philosophers to discuss topics such as mystery in mathematics and faith, and Gödel's Incompleteness Theorems. Finally, to break things up a little bit, we spent one evening with students playing a variety of board games (especially games emphasizing logic and strategy).

In addition to scheduled events during the residential part of the program, we asked students to do some reading and writing at the intersection of mathematics, philosophy, and theology. At the end of the residential program, we handed students copies of Flatland and a couple of chapters from *Mathematics Through the Eyes of Faith* [1], along with prompts for essays. These readings and writings served as a sort of capstone to the program, giving students the chance to process and respond to some new material in light of all that they had learned during the residential portion of the program.

Outcomes and Feedback

All told, 19 students attended the Mystery of the Infinite course, and everyone survived. Of those students, 14 passed the course with a C or better (with more A's than any other grade). Of those who did not achieve a C, most were students who struggled from the beginning of the course with the speed and with the transition to abstract material. We ultimately counseled some of these students to forget about the grade and focus on getting as much out of the program as was reasonable for them (counsel that was uniformly well received).

Perhaps a better indicator of the success of the program, however, is the feedback left by students. The most common remark in the feedback was that students would have liked more time to process the material; this was not too shocking, as it was clear by the end of the residential part of the program that even the best students were exhausted. However, each student was also very positive about the program overall. For example, see the comments below:

- "The workload and pacing were enough to push everyone to their limits, and was done well."

- "Before the program I was a little unsure if I was going to enjoy it, but after I feel like I don't want to even leave, I had such a fantastic time."
- "Before I was worried about a systematic learning experience, however it proved to be very personal and hands-on."
- "The learning atmosphere was amazing!"
- "We learned about math and how God was related. It was more fun than I expected."
- "I thought it would be more rigid. It was not, I felt very supported and encouraged when I was confused."
- "This was a lot more interesting than what I was used to."
- "It was an amazing experience being surrounded by other Christians and getting to learn more about theology and math related to Christianity."

A further measure of the success of the Summer Scholars Program, as a whole, is its proficiency at recruiting students. Of the approximately 60 students who have participated during the first two years of the program, 14 have ultimately enrolled at Eastern (keep in mind that many of the students who have participated are only now entering their senior year of high school, so the proportion is likely to increase further).

Challenges

To any who are interested in developing a similar program, we can now confidently say that it is not a small undertaking. Indeed, we encountered numerous challenges along the way (some of which are still being worked out); these challenges can be broadly classified as length of program, cost, a text, and personnel.

As noted above, the length of the program is a feature which significantly affects the student's experience. Several factors went into settling on a 9 day residential program. The first of these is cost, which must be kept low enough to attract students. We were able to obtain some outside funding for scholarships, but students still paid approximately $1,800 for the program; as this included all expenses (including the trip to the Museum of Mathematics) and most students earned 3 college credits, this cost is not unreasonable, but it does still limit the pool of applicants. To run the program for a longer period would have necessitated raising costs further.

On the other end of things, the length of the program was shaped by the amount of material to be covered. Even with nine days, we were hurried to complete the course. We also had to be very careful to plan the number of contact hours with students so as to ensure that state mandated minimums were being satisfied.

Moving on, we quickly realized that we would need to provide students with a text. Again, cost constrained us here, and we ultimately decided to write our own text. In many ways, this turned out to be a blessing, as we were able to integrate a number of faith perspectives and philosophical asides holistically into the text. At present, we have 129 pages of TeX-ed material, complete with a large number of exercises, and we plan to continue refining the material over the coming years (I plan to use it for my Discrete Math class in the spring of 2016). Ultimately, we hope that such a text might find a market within faith-based schools.

With regard to personnel, we were blessed with some fantastic administrative support and leadership. They took care of many of the logistical details (advertising, enrollment, dorms, meals, transportation, etc), and helped the students to feel at home within the first moments of their arrival at Eastern. We were also able to recruit some fantastic teaching assistants from the ranks of Eastern Math majors (and even a recent alum). The TA's spent nearly every waking hour with students, while also managing to prepare for problem sessions and grade homework. Their importance to the program cannot be overstated.

Finally, and still on the topic of personnel, the program also requires a large commitment from the faculty. My colleague and I put untold hours into planning the course and (especially) writing the text. During the residential portion of the program we put in full days with the students, sometimes staying late into the night. Even after the completion of the program, we had to grade exams and essays. Despite this work, monetary compensation was minimal (there is some hope that pay will grow as more students are attracted to the program, and that workloads will decrease as the program grows more established). All that said, it was a fantastic experience to work with these young students, giving them a glimpse into how rich mathematics can be.

Future Development and Conclusion

With the overall success of the Mystery of the Infinite course last summer, we are planning to continue offering the course on a biannual basis (a number of students participate in the SSP for two summers in a row, so we want to keep the offerings fresh). That said, we've learned quite a bit even in the first incarnation, and we intend to continue developing the course. In particular, we see significant room for improvement in the intensity of the program, in our staffing, and in the text.

As noted in the previous section, most of our students found the program to be very intense and would have liked more time. One possible change, therefore, would be to run the program for more days, provided that we can keep the cost low enough to continue attracting students. Alternately, we may adjust the syllabus; in particular, the integrity of the course could probably be maintained without covering equivalence relations (from my conversations with faculty at other institutions, this topic is often omitted from a Discrete Math course). More radically, we could also choose to drop the explicit connection to our Discrete Mathematics course, instead offering students general credit. This would have the disadvantage of likely not transferring to schools other than Eastern, but it would let us relax expectations quite a bit and focus on making the program interesting and enjoyable. Apparently, this approach was taken in a similar program at Taylor University, and the following years actually saw a (somewhat counterintuitive) boost in student interest and attendance.

With regard to staffing, Dr. Huddell and I are cognizant that we are two white males, and we don't want to send false signals about who is qualified college level mathematics. As such, we are hoping to recruit another professor to add some diversity to the program faculty. We also plan to continue investing time and energy in the textbook. With more time, we expect to expand up on the faith integration, to refine and add to the exercises, to provide solutions to at least some of the exercises, and, possibly, to add more content.

Moving beyond the particular Mystery of the Infinite course, we see an opportunity to develop more mathematics courses for the Summer Scholars Program. For example, a course on the basics of Non-Euclidean geometry would be a very interesting follow up for students who have recently taken a high school course in Euclidean geometry (and, the development of Non-Euclidean geometry again offers a host of interesting points for faith integration). We also see some potential in courses on Dynamical Systems or Knot Theory.

All in all, the Mystery of the Infinite does seem to have been a success, and it looks like it will continue to improve in coming years (I'm excited to see where it goes!). That said, I close with a word to the reader. If you have been involved in developing or running similar programs and would like to share ideas, we would be grateful to learn from your experience and to share more of ours—please get in touch. On other end of things, if you are working to get a program started, we would again welcome you to contact us with any questions. Finally, if you know of any students who might be interested in participating in the program, please send them our way! Information about the Summer Scholars Program can be found at templetonhonorscollege.com/summer, or by contacting the author.

References

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